

CHAPTER 2

Performance at International Benchmarks

The TIMSS 1999 international benchmarks delineate performance of the top 10 percent, top quarter, top half, and lower quarter of students in the entities participating in the study. To help interpret the achievement results, Chapter 2 describes eighth-grade mathematics achievement at each of these benchmarks together with examples of the types of items typically answered correctly by students performing at the benchmark.



To provide an idea of the mathematics understandings and skills displayed by students performing at different levels on the TIMSS mathematics achievement scale, TIMSS described performance at four international benchmarks. The TIMSS 1999 international benchmarks delineate performance of the top 10 percent, top quarter, top half, and lower quarter of students in the countries participating in the TIMSS 1999 study. (The benchmarks were set at the 90th, 75th, 50th, and 25th percentiles, respectively.)

As states and school districts spend time and energy on improving students' mathematics achievement, it is important that educators, curriculum developers, and policy makers understand what students know and can do in mathematics, and what areas, concepts, and topics need more focus and effort. To help interpret the range of achievement results for the TIMSS 1999 Benchmarking participants presented in Chapter 1, this chapter describes eighth-grade mathematics achievement at each of the TIMSS 1999 international benchmarks, explaining the types of mathematics understandings and skills typically displayed by students performing at the benchmarks. The benchmark descriptions are presented together with examples of the types of mathematics test questions typically answered correctly by students reaching the benchmark. Appendix D contains the descriptions of the understandings and skills assessed by each item in the TIMSS 1999 assessment at each benchmark.¹

For each of the example test questions, the percentages of correct responses are provided for selected countries as well as for the jurisdictions participating in the TIMSS 1999 Benchmarking project. The countries and Benchmarking jurisdictions are presented in descending order, with those performing highest shown first. The countries included for purposes of comparison are the United States as well as a dozen European and Asian countries of interest. These include several high-performing European countries (Belgium (Flemish), the Czech Republic, the Netherlands, and the Russian Federation), countries that are major economic trading partners of the United States (Canada, England, and Italy), and the top-scoring Asian countries of Chinese Taipei, Hong Kong, Japan, Korea, and Singapore.

Presented previously in Chapter 1, Exhibit 1.4 shows the percentages of students in each participating entity reaching each international benchmark – Top 10%, Upper Quarter, Median, and Lower Quarter. If an entity had high average achievement in mathematics and a large percentage of its students at or above the upper benchmarks, this indicates that the students are concentrated among the highest-achieving

¹ For a detailed description of the items and benchmarks for TIMSS 1995 at fourth and eighth grades and how they compare to the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics*, see Kelly, D.L., Mullis, I.V.S., and Martin, M.O., *Profiles of Student Achievement in Mathematics at the TIMSS International Benchmarks: U.S. Performance and Standards in an International Context*, Chestnut Hill, MA: Boston College.

students internationally. For example, top-performing Singapore had nearly half (46 percent) of its students reaching the Top 10% Benchmark and three-fourths (75 percent) reaching the Upper Quarter Benchmark – the point on the scale that typically only 25 percent of the students would be expected to reach if achievement were distributed equally from country to country. Most of the Singaporean students (93 percent) reached the Median Benchmark. Performance in the United States was closer to the distribution that might be expected if achievement were distributed the same from country to country: nine percent of the students reached the Top 10% Benchmark, 28 percent reached the Top Quarter Benchmark, and 61 percent reached the Median Benchmark.

The analysis of performance at these benchmarks in mathematics suggests that three primary factors appeared to differentiate performance at the four levels:


- The mathematical operation required
- The complexity of the numbers or number system
- The nature of the problem situation.

For example, there is evidence that students performing at the lower end of the scale could add, subtract, and multiply whole numbers. In contrast, students performing at the higher end of the scale solved non-routine problems involving relationships among fractions, decimals, and percents; various geometric properties; and algebraic rules.

How Were the Benchmark Descriptions Developed?

To develop descriptions of achievement at the TIMSS 1999 international benchmarks, the International Study Center used the scale anchoring method. Scale anchoring is a way of describing students' performance at different points on the TIMSS 1999 achievement scale in terms of the types of items they answered correctly. It involves an empirical component in which items that discriminate between successive points on the scale are identified, and a judgmental component in which subject-matter experts examine the content of the items and generalize to students' knowledge and understandings.

For the scale anchoring analysis, the results of students from all the TIMSS 1999 countries were pooled, so that the benchmark descriptions refer to all students achieving at that level. (That is, it does not matter which country the students are from, only how they performed on the test.) Certain criteria were applied to the TIMSS 1999 achievement scale results



to identify the sets of items that students reaching each international benchmark were likely to answer correctly and those at the next lower benchmark were unlikely to answer correctly.² The sets of items thus produced represented the accomplishments of students reaching each benchmark and were used by a panel of subject-matter experts from the TIMSS countries to develop the benchmark descriptions.³ The work of the panel involved developing a short description for each item of the mathematical understandings demonstrated by students answering it correctly, summarizing students' knowledge and understandings across the set of items for each benchmark to provide more general statements of achievement, and selecting example items illustrating the descriptions.

How Should the Descriptions Be Interpreted?

In general, the parts of the descriptions that relate to the understanding of mathematical concepts or familiarity with procedures are relatively straightforward. It needs to be acknowledged, however, that the cognitive behavior necessary to answer some items correctly may vary according to students' experience. An item may require only simple recall for a student familiar with the item's content and context, but necessitate problem-solving strategies from one unfamiliar with the material. Nevertheless, the descriptions are based on what the panel believed to be the way the great majority of eighth-grade students could be expected to perform.

It also needs to be emphasized that the descriptions of achievement characteristic of students at the international benchmarks are based solely on student performance on the TIMSS 1999 items. Since those items were developed in particular to sample the mathematics domains prescribed for this study, neither the set of items nor the descriptions based on them purport to be comprehensive. There are undoubtedly other mathematics curriculum elements on which students at the various benchmarks would have been successful if they had been included in the assessment.

Please note that students reaching a particular benchmark demonstrated the knowledge and understandings characterizing that benchmark as well as those characterizing the lower benchmarks. The description of achievement at each benchmark is cumulative, building on the description of achievement demonstrated by students at the lower benchmarks.

² For example, for the Top 10% Benchmark, an item was included if at least 65 percent of students scoring at the scale point corresponding to this benchmark answered the item correctly and less than 50 percent of students scoring at the Upper Quarter Benchmark answered it correctly. Similarly, for the Upper Quarter Benchmark, an item was included if at least 65 percent of students scoring at that point answered the item correctly and less than 50 percent of students at the Median Benchmark answered it correctly.

³ The participants in the scale anchoring process are listed in Appendix E.

Finally, it must be emphasized that the descriptions of the international benchmarks are one possible way of beginning to examine student performance. Some students scoring below a benchmark may indeed know or understand some of the concepts that characterize a higher level. Thus, it is important to consider performance on the individual items and clusters of items in developing a profile of student achievement in each participating entity.

Several example items are included for each benchmark to complement the descriptions by giving a more concrete notion of the abilities students demonstrated. Each example item is accompanied by the percentage of correct responses for each TIMSS 1999 Benchmarking participant. Percentages are also provided for selected countries, as is the international average for all 38 countries that participated in TIMSS 1999. In general, the several entities scoring highest on the overall test also scored highest on many of the example items. Not surprisingly, this was true for items assessing a range of performance expectations – recall, ability to carry out routine procedures, and ability to solve routine and non-routine problems. The TIMSS 1999 results support the premise that successful problem solving is grounded in mastery of more fundamental knowledge and skills.

Item Examples and Student Performance

The remainder of this chapter describes each benchmark and presents three to five example items illustrating what students know and can do at that level. The correct answer is circled for multiple-choice items. For open-ended items, the answers shown exemplify the types of student responses that were given full credit. The example items are ones that students reaching each benchmark were likely to answer correctly, and they represent the types of items used to develop the description of achievement at that benchmark.⁴

⁴ Some of the items used to develop the benchmark descriptions are being kept secure to measure achievement trends in future TIMSS assessments and are not available for publication.

Achievement at the Top 10% Benchmark

Exhibit 2.1 describes performance at the Top 10% Benchmark. Students reaching this benchmark demonstrated the ability to organize information in problem-solving situations and to apply their understanding of mathematical relationships. They typically demonstrated success on the knowledge and skills represented by this benchmark, as well as those demonstrated at the three lower benchmarks.

Example Item 1 in Exhibit 2.2 illustrates the type of measurement item a student performing at the Top 10% Benchmark generally answered correctly. As can be seen, students had to apply their knowledge of the area of rectangles and inscribed shapes to solve a two-step problem about the area of a garden path. The international average for this item was 42 percent correct, indicating that this was a relatively difficult item for eighth graders around the world. Nevertheless, more than two-thirds of the students answered the item correctly in Hong Kong, Singapore, Japan, Chinese Taipei, and Korea. Among the Benchmarking participants, eighth graders in the Naperville School District did as well as their counterparts in the high-performing Asian countries, with 69 percent answering correctly. Generally, however, students in the United States – in the country as a whole and in the Benchmarking entities – performed relatively less well than students internationally on measurement questions involving relationships between shapes. No other Benchmarking entity performed significantly above the international average on this test question, and students in six Benchmarking entities and in the United States overall performed significantly below the international average. On average internationally, more than 20 percent of students chose Option A, solving for the area of the larger rectangle rather than that of the path. Option C was an equally popular distracter, selected by more than 20 percent of students internationally.

Unlike students performing at lower benchmarks, students reaching the Top 10% Benchmark typically could correctly answer multistep word problems. Example Item 2 in Exhibit 2.3 requires students to select relevant information from two advertisements to solve a complex multistep word problem involving decimals. Given the price for each issue of a magazine and a certain number of free issues, students were asked to calculate which of the two magazine subscriptions was the less expensive for 24 issues. Students received full credit if they showed correct calculations for at least one of the subscriptions, identified the less expensive magazine, and calculated the difference between the two

• Top 10% Benchmark

Summary

Students can organize information, make generalizations, and explain solution strategies in non-routine problem solving situations. They can organize information and make generalizations to solve problems; apply knowledge of numeric, geometric, and algebraic relationships to solve problems (e.g., among fractions, decimals, and percents; geometric properties; and algebraic rules); and find the equivalent forms of algebraic expressions.

Students can organize information in problem-solving situations. They can select and organize information from two sources to solve a complex word problem involving decimals and organize information to solve a multi-step word problem involving whole numbers.

Students can correctly order the four basic operations in computing with decimals and fractions. Students use their understanding of fractions and decimals in multi-step problem situations. They can solve a problem involving both addition and subtraction of simple common fractions and a problem involving multiplication and subtraction of decimals. They can solve word problems involving fractions and decimals which require analysis of the verbal relations described. They can order a set of decimal fractions of up to three decimal places and can identify the pair of numbers satisfying given conditions involving ordering integers, decimals, and fractions. They can solve a time-distance-rate problem involving decimals and the conversion of minutes to seconds. They can work with part-whole ratios and can solve word problems to find the percent change.

Students can apply their knowledge of measurement in more complex problem situations. They can solve problems involving area and perimeter of rectangles and area of inscribed triangles. They apply knowledge of properties of squares to solve multi-step word problems and draw a new rectangle based on a given rectangle and express the ratio of their areas. They can relate different units of time and apply their knowledge of the number of milliliters in a liter to solve a word problem. They recognize that precision of measurement is related to the size of the unit of measurement.


Students can use their knowledge of angles – overlapping and measures of angles in quadrilaterals – to solve problems. They can use their knowledge of congruent and similar triangles to solve problems concerning corresponding parts. They can identify the coordinates of a point on a line given the coordinates of two other points on the line and locate a point on a number line given its distance from two other points on the line. They can identify the image of a triangle under a rotation in a plane.

Students can use proportion to find missing values in a table. Students can identify an equivalent form of a linear inequality involving a fraction. Students can recognize properties of number operations represented in symbolic form. They can solve a multi-step word problem in which there are two unknowns.

Given the first several terms in pictorial form, that grow in either one or two dimensions, students can make generalizations to find terms in the sequences (e.g. 51st), and they can explain the process used to find those terms.

90th Percentile: 616

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.



subscriptions. With an international average of 24 percent correct (for full credit), this item was among the most difficult in TIMSS 1999. Singapore, Korea, and Chinese Taipei were the only countries where the majority of the students answered correctly. The best performance by a Benchmarking entity was in Naperville, where 41 percent of the eighth graders answered correctly. Students in the First of World Consortium (36 percent) and Montgomery County (35 percent) also performed significantly above the international average.

Students reaching the Top 10% Benchmark exhibited an understanding of the properties of similar triangles, as shown by Example Item 3 (see Exhibit 2.4). Given two angle measurements, the length of a side of a triangle, and the dimensions of a second similar triangle, students needed to find the length of an unlabeled side of the first triangle. Internationally, most eighth-grade students had not mastered the concept of proportionality of corresponding sides or could not solve the resulting equation; only 37 percent, on average, answered the question correctly. In comparison, top-performing Korea had 70 percent correct responses. Among the TIMSS 1999 countries, only in Korea, Japan, Singapore, Hong Kong, Chinese Taipei, and Belgium (Flemish) did at least half the students answer correctly. In the Benchmarking jurisdictions, correct responses were provided by more than half the eighth graders in Naperville (56 percent) and the First in the World Consortium (52 percent).

The eighth-grade students reaching the Top 10% Benchmark typically were able to apply a generalization to solve a sequence problem like the one shown in Example Item 4 in Exhibit 2.5. In this algebra problem, given the initial terms in a sequence and the 50th term of that sequence, students generalized to find the 51st term. Even though results are presented only for Part C, this problem was presented in three parts, A, B, and C. To provide some scaffolding, parts A and B asked students to indicate how many circles would be in the 5th and 7th figures, respectively, if the pattern were extended. On average internationally, 65 percent of the students answered Part A correctly and 54 percent successfully extended the sequence to the 7th figure in Part B.

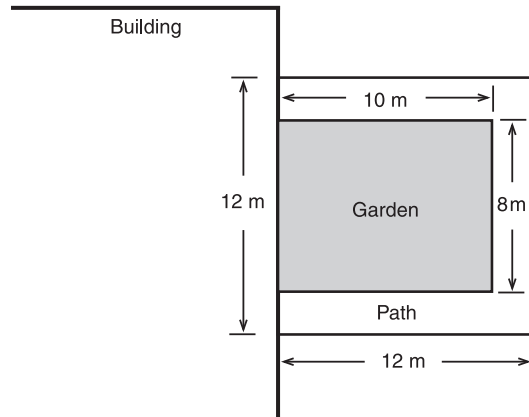
To receive full credit for Part C, students had to show or explain how they arrived at their answer by providing a general expression or an equation and by calculating the correct number of circles for the 51st figure. Internationally on average, 30 percent of the students received full credit for their responses. In comparison, about two-thirds of the students in Korea, Chinese Taipei, Japan, and Singapore received full credit. Although eighth graders in six Benchmarking entities – First in

the World, Naperville, the Michigan Invitational Group, Montgomery County, the Academy School District, and Oregon – performed significantly above the international average, their performance was below that of the top performers, ranging from 54 to 39 percent correct. Most students added the sequence number to the number of circles in the preceding figure: $1275 + 51 = 1326$. Very few calculated the answer by a general expression: $n(n+1)/2$ or $51(52)/2$ (although 13 percent of the Dutch students did so).

Content Area: Measurement

Description: Finds the area between two rectangles when one is inside the other and their sides are parallel.

A rectangular garden that is next to a building has a path around the other three sides, as shown.



What is the area of the path?

- A. 144 m²
- B. 64 m²
- C. 44 m²
- D. 16 m²

	Overall Percent Correct	
Hong Kong, SAR †	79 (2.0)	▲
Singapore	78 (2.6)	▲
Japan	74 (1.9)	▲
Chinese Taipei	73 (2.1)	▲
Naperville Sch. Dist. #203, IL	69 (4.0)	▲
Korea, Rep. of	67 (1.7)	▲
Netherlands †	57 (4.4)	▲
First in the World Consort., IL	56 (5.9)	●
Canada	51 (3.0)	●
Belgium (Flemish) †	51 (2.2)	▲
Montgomery County, MD ²	46 (4.6)	●
Italy	45 (2.7)	●
Oregon	42 (3.9)	●
Michigan Invitational Group, MI	41 (4.4)	●
Czech Republic	40 (3.5)	●
England †	40 (3.3)	●
Illinois	40 (3.4)	●
Project SMART Consortium, OH	39 (5.2)	●
Maryland	38 (2.8)	●
Russian Federation	38 (3.2)	●
Fremont/Lincoln/WestSide PS, NE	38 (3.6)	●
SW Math/Sci. Collaborative, PA	38 (4.1)	●
<i>Texas</i>	38 (3.8)	●
Massachusetts	35 (3.0)	●
Academy School Dist. #20, CO	35 (4.4)	●
Guilford County, NC ²	35 (5.7)	●
Indiana †	34 (3.1)	●
Idaho	34 (2.8)	●
Connecticut	33 (3.9)	●
Michigan	33 (3.3)	●
United States	33 (1.6)	▼
Delaware Science Coalition, DE	32 (3.1)	●
Missouri	31 (3.0)	▼
<i>Pennsylvania</i>	30 (2.8)	▼
South Carolina	30 (4.1)	●
North Carolina	29 (3.2)	▼
Rochester City Sch. Dist., NY	27 (4.4)	▼
Chicago Public Schools, IL	26 (5.9)	●
Jersey City Public Schools, NJ	25 (3.5)	▼
Miami-Dade County PS, FL	21 (3.0)	▼
International Avg. (All Countries)	42 (0.4)	
Participant average significantly higher than international average	▲	
No statistically significant difference between participant average and international average	●	
Participant average significantly lower than international average	▼	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Data Representation, Analysis and Probability

Description: Selects relevant information from two advertisements to solve a complex word problem involving decimals.

Chris plans to order 24 issues of a magazine. He reads the following advertisements for two magazines. *Ceds* are the units of currency in Chris' country.

Teen Life Magazine

24 issues
First four issues FREE
The rest
3 *ceds* each.

Teen News Magazine

24 issues
First six issues FREE
The rest
3.5 *ceds* each.

Which magazine is the least expensive for 24 issues? How much less expensive? Show your work.

$$\begin{array}{r} \text{Teen Life} = 20 \\ \times 3 \\ \hline 60 \text{ ceds} \\ 24 = 60 \text{ ceds} \end{array} \qquad \begin{array}{r} \text{Teen News} = 18 \\ \times 3.5 \\ \hline 90 \\ 540 \\ \hline 63.0 \\ \text{ceds} \\ 24 = 63 \text{ ceds} \end{array}$$

Teen Life is less expensive by 3 *ceds*.

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct
Singapore	57 (2.1) ▲
Korea, Rep. of	52 (1.5) ▲
Chinese Taipei	50 (1.8) ▲
Belgium (Flemish) †	42 (1.7) ▲
Naperville Sch. Dist. #203, IL	41 (2.6) ▲
Japan	39 (1.5) ▲
First in the World Consort., IL	36 (2.9) ▲
Montgomery County, MD ²	35 (2.8) ▲
Hong Kong, SAR †	34 (1.8) ▲
Czech Republic	34 (2.5) ▲
Canada	32 (1.8) ▲
Connecticut	32 (2.7) ●
Texas	31 (4.0) ●
Russian Federation	30 (2.4) ●
Project SMART Consortium, OH	30 (3.5) ●
Indiana †	29 (3.5) ●
Massachusetts	29 (2.7) ●
Michigan Invitational Group, MI	29 (2.2) ●
Academy School Dist. #20, CO	27 (2.5) ●
Italy	27 (1.7) ●
Jersey City Public Schools, NJ	27 (4.4) ●
SW Math/Sci. Collaborative, PA	27 (3.2) ●
Guilford County, NC ²	26 (2.4) ●
Pennsylvania	26 (2.9) ●
United States	26 (1.4) ●
Michigan	26 (2.2) ●
Illinois	25 (3.1) ●
Netherlands †	25 (2.7) ●
South Carolina	25 (2.2) ●
Idaho	25 (2.8) ●
North Carolina	23 (2.2) ●
Maryland	23 (2.1) ●
Oregon	22 (2.5) ●
Delaware Science Coalition, DE	22 (3.8) ●
Missouri	21 (1.6) ●
Fremont/Lincoln/WestSide PS, NE	20 (3.7) ●
Chicago Public Schools, IL	19 (3.4) ●
England †	17 (1.9) ▼
Rochester City Sch. Dist., NY	15 (2.3) ▼
Miami-Dade County PS, FL	11 (2.3) ▼
International Avg. (All Countries)	24 (0.3)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered fully correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

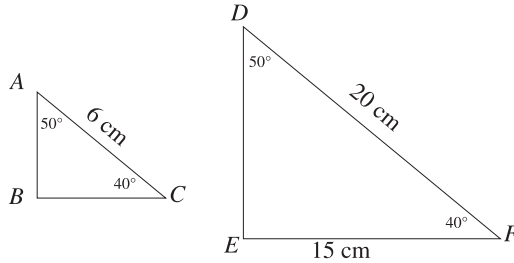
² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Geometry

Description: Uses properties of similar triangles to find the length of a corresponding side.

The figure represents two similar triangles. The triangles are not drawn to scale.



In the actual triangle *ABC*, what is the length of side *BC*?

- A 3.5 cm
- B 4.5 cm**
- C 5 cm
- D 5.5 cm
- E 8 cm

	Overall Percent Correct
Korea, Rep. of	70 (1.9) ▲
Japan	68 (1.9) ▲
Singapore	64 (2.7) ▲
Hong Kong, SAR [†]	56 (2.2) ▲
Naperville Sch. Dist. #203, IL	56 (3.6) ▲
First in the World Consort., IL	52 (4.7) ▲
Chinese Taipei	52 (2.3) ▲
Belgium (Flemish) [†]	50 (3.2) ▲
Academy School Dist. #20, CO	46 (4.2) ●
Guilford County, NC²	45 (5.4) ●
Netherlands [†]	44 (3.1) ●
SW Math/Sci. Collaborative, PA	43 (2.9) ●
Texas	43 (5.0) ●
Montgomery County, MD²	42 (3.6) ●
Russian Federation	41 (2.7) ●
Connecticut	40 (3.8) ●
Illinois	40 (2.2) ●
Idaho	39 (4.2) ●
Massachusetts	38 (2.8) ●
North Carolina	38 (3.4) ●
Indiana[†]	38 (3.7) ●
Michigan	37 (3.3) ●
South Carolina	37 (2.6) ●
Delaware Science Coalition, DE	37 (4.1) ●
Oregon	36 (3.9) ●
United States	36 (1.6) ●
Maryland	35 (2.5) ●
Michigan Invitational Group, MI	35 (4.0) ●
Canada	35 (2.2) ●
England [†]	34 (2.7) ●
Chicago Public Schools, IL	32 (4.5) ●
Jersey City Public Schools, NJ	32 (3.7) ●
Miami-Dade County PS, FL	32 (3.2) ●
Czech Republic	32 (2.5) ●
Pennsylvania	32 (2.8) ●
Project SMART Consortium, OH	31 (4.4) ●
Italy	29 (2.4) ▼
Fremont/Lincoln/WestSide PS, NE	29 (5.7) ●
Missouri	27 (3.0) ●
Rochester City Sch. Dist., NY	26 (4.0) ●
International Avg. (All Countries)	37 (0.4)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

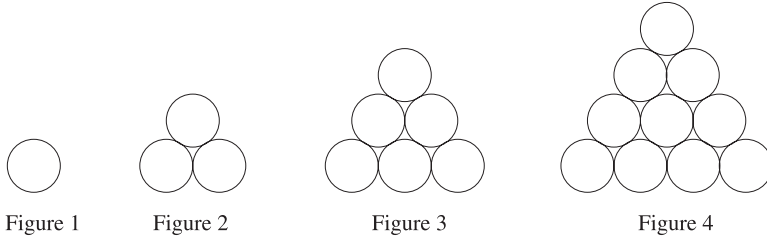
² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Algebra

Description: Given the initial terms in a sequence and, for example, the 50th term of that sequence, generalizes to find the next term.

The figures show four sets consisting of circles.



- a) Complete the table below. First, fill in how many circles make up Figure 4. Then, find the number of circles that would be needed for the 5th figure if the sequence of figures is extended.

Figure	Number of circles
1	1
2	3
3	6
4	10
5	15

- b) The sequence of figures is extended to the 7th figure. How many circles would be needed for Figure 7?

Answer: 28

- c) The 50th figure in the sequence contains 1275 circles. Determine the number of circles in the 51st figure. Without drawing the 51st figure, explain or show how you arrived at your answer.

Because it is the 51st figure you have to add 51 to the base of the figure before it

$$\begin{array}{r} 1275 \\ + 51 \\ \hline 1326 \end{array}$$

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct	
Korea, Rep. of	70 (1.2) ▲	
Chinese Taipei	68 (1.5) ▲	
Japan	66 (1.6) ▲	
Singapore	65 (2.4) ▲	
Hong Kong, SAR [†]	57 (2.0) ▲	
First in the World Consort., IL	54 (4.2) ▲	
Naperville Sch. Dist. #203, IL	53 (3.6) ▲	
Michigan Invitational Group, MI	53 (3.7) ▲	
Netherlands [†]	48 (3.0) ▲	
Montgomery County, MD²	46 (3.8) ▲	
Belgium (Flemish) [†]	44 (1.7) ▲	
Academy School Dist. #20, CO	44 (4.1) ▲	
Canada	43 (2.2) ▲	
Massachusetts Connecticut	40 (3.5) ●	
	39 (3.5) ●	
Oregon	39 (2.7) ▲	
Michigan	39 (3.2) ●	
Project SMART Consortium, OH	39 (4.5) ●	
Indiana [†]	37 (3.8) ●	
Guilford County, NC²	36 (2.8) ●	
<i>Texas</i>	36 (4.0) ●	
SW Math/Sci. Collaborative, PA	35 (3.5) ●	
Fremont/Lincoln/WestSide PS, NE	35 (3.6) ●	
Delaware Science Coalition, DE	35 (3.3) ●	
England [†]	35 (2.5) ●	
United States	34 (1.3) ▲	
Pennsylvania	34 (2.7) ●	
South Carolina	34 (2.7) ●	
Czech Republic	34 (2.5) ●	
Jersey City Public Schools, NJ	33 (4.8) ●	
Idaho	32 (2.9) ●	
North Carolina	32 (2.6) ●	
Maryland	30 (2.8) ●	
Missouri	30 (1.8) ●	
Illinois	30 (2.4) ●	
Russian Federation	27 (2.0) ●	
Italy	24 (1.8) ▼	
Miami-Dade County PS, FL	20 (2.5) ▼	
Chicago Public Schools, IL	19 (3.2) ▼	
Rochester City Sch. Dist., NY	17 (3.1) ▼	
International Avg. (All Countries)	30 (0.3)	
Participant average significantly higher than international average	▲	
No statistically significant difference between participant average and international average	●	
Participant average significantly lower than international average	▼	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Significance tests adjusted for multiple comparisons

* The item was answered fully correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details)

[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Achievement at the Upper Quarter Benchmark

Exhibit 2.6 describes performance at the Upper Quarter Benchmark. Eighth-grade students performing at this level applied their mathematical knowledge and understandings in a wide variety of relatively complex problem situations. For example, they demonstrated facility with fractions in various formats, as illustrated by Example Item 5 shown in Exhibit 2.7. This item required students to shade squares in a rectangular grid to represent a given fraction. Since the grid is divided into squares that are a multiple of the fraction's denominator, more than one step is required to solve the problem. Internationally, about half the students (49 percent on average) were able to shade in nine of the 24 squares to represent $\frac{3}{8}$ of the region. Eighty percent or more of the students in Singapore, Hong Kong, Belgium (Flemish), Korea, and Chinese Taipei answered the question correctly. No Benchmarking entities performed that well, but students in the First in World Consortium, Naperville, the Michigan Invitational Group, and Massachusetts performed significantly above the international average.

Example Item 6 is a proportional reasoning word problem that students at the Upper Quarter Benchmark typically answered correctly (see Exhibit 2.8). Given the number of magazines sold by each of two boys and the total amount of money made from the sales, students were to calculate how much money one of the boys made by selling his 80 magazines. On average, 44 percent of students internationally answered this question correctly. In Singapore and Chinese Taipei at least three-quarters of the students answered correctly. No Benchmarking participant performed significantly above the international average, and students in Maryland, the Michigan Invitational Group, the Chicago Public Schools, the Rochester City School District, and the Miami-Dade County Public Schools performed significantly below the international average.

Students reaching the Upper Quarter Benchmark generally were able to apply knowledge of geometric properties. In Example Item 7 in Exhibit 2.9, students needed to use their knowledge of the properties of parallelograms and rectangles to solve for the area of the rectangle (dimensions not labeled) that was part of a different figure with given dimensions. Three-quarters or more of the students in Singapore, Japan, Hong Kong, Korea, and Chinese Taipei answered the item correctly. Internationally, however, less than half the eighth-grade students (43 percent on average) did so. The United States performed

significantly below the international average, as did eight of the Benchmarking entities: North Carolina, South Carolina, Missouri, the Delaware Science Coalition, and the public school systems in Jersey City, Chicago, Miami-Dade, and Rochester.

Example Item 8 shown in Exhibit 2.10 asks students for the number of triangles of a given dimension needed to cover a rectangle of given dimensions. The international average on this item was 46 percent correct. Many students (approximately 29 percent internationally) incorrectly chose Option A, which is half the number of required triangles needed to fill the rectangle but just enough to cover the perimeter. Japanese students had the highest performance on this item, with 80 percent answering correctly. About two-thirds or more of the students in Korea, Hong Kong, Singapore, Belgium (Flemish), and the Netherlands answered the item correctly. Performance among the Benchmarking participants ranged from 62 percent correct responses in Naperville to 30 percent in Miami-Dade. The United States as a whole performed at about the international average, and most of the Benchmarking jurisdictions performed similarly.

Unlike students at lower benchmarks, those reaching the Upper Quarter Benchmark typically could solve simple linear equations. As illustrated by Example Item 9 in Exhibit 2.11, for example, students successfully solved for the value of x in a linear equation involving the variable on both sides of the equation. Eighty percent or more of the students in Japan, Hong Kong, and Korea answered this item correctly. Even though the United States did relatively well in algebra (see Chapter 3), this problem posed difficulties for students in the Benchmarking entities. Naperville (72 percent) and First in the World (61 percent) were the only Benchmarking participants that performed significantly above the international average of 44 percent correct responses. The United States performed below average (34 percent) on this question, as did students in 11 of the Benchmarking entities.

• Upper Quarter Benchmark

Summary

Students can apply their understanding and knowledge in a wide variety of relatively complex situations. They can order, relate and compute with fractions and decimals to solve word problems; solve multi-step word problems involving proportions with whole numbers; solve probability problems; use knowledge of geometric properties to solve problems; identify and evaluate algebraic expressions and solve equations with one variable.

Students demonstrate some facility with fractions and decimals through computation, ordering, rounding, and use in word problems. They can recognize equivalent fractions, add, subtract, multiply and divide fractions with unlike denominators, and correctly order operations. They can identify the smallest decimal from a set of decimals with differing number of places and provide a fraction that is less than a given fraction. They can solve word problems involving multiplication and division of whole numbers and fractions and use pictorial representations of fractions in solving problems. They can identify the fraction of an hour representing a given time interval and identify fractions representing the comparison of part to whole, given each of two parts in a word problem setting.

Students can select the correct rounding of a number involving four decimal places, identify the decimal that is between two decimals given in hundredths, and solve a word problem that involves multiplying a decimal in thousandths by a multiple of a hundred. They can produce an example of a number that would round to a given value. Given a length rounded to the nearest centimeter, they can identify an example of the actual length expressed to one decimal place. Students can identify the ratio expressing a given whole number comparison in a word problem and recognize the effect of adding the same amount to both terms of a ratio. They can estimate products of whole numbers to solve problems. They can solve multi-step word problems involving proportions with whole numbers.

Students demonstrate their understanding of measurement in several settings. They can compare volumes by visualizing and counting cubes. They can calculate the areas of rectangles contained in diagrams of combined shapes. Given the start time and the duration of an event expressed as a fraction of an hour, they can determine the end time. They can estimate the distance between two points on a map, given the scale, and can read unlabeled tick marks on a scale.

Students can use basic properties of triangles, properties of angles on a straight line, and knowledge of symmetry to find the measures of angles. They can identify the angle in a diagram that represents the best estimate of a given measure and recognize that internal angles on a transversal are supplementary. They can visualize the center of a rotation for a two-dimensional figure, the arrangement of faces of a cube when shown its net, and the number of triangles of given dimensions needed to cover a given rectangle. They can identify false statements about congruent triangles and the properties of rectangles.

Students understand elementary concepts of probability, including independent events. They can solve simple problems involving the relationship between successful and unsuccessful outcomes and probabilities. They also recognize that when outcomes are expressed as fractions of a whole, the least likely outcome corresponds to the smallest fraction. They can extrapolate from a graph and determine the number of values on the horizontal axis of a line graph that correspond to a given value on the vertical axis. On a given graph, students can interpolate to find a value between gradations on one axis matching a given value on the other axis.

Students can recognize that multiplication can represent repeated addition. They can identify the algebraic equation corresponding to a verbal description. They can select a simple, multiplicative expression in one variable that is positive for all negative values of the variable. They can substitute numbers for variables to evaluate an expression, and subtract fractions represented algebraically with the same numeric denominator.

Students can solve a linear equation with or without parentheses. They can identify the linear equation that describes the relationship between two variables given in a table of values and select the formula satisfied by the given values of the variables. They can identify the relationship between the first and second terms in a set of ordered pairs.

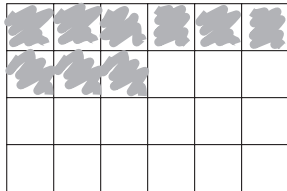
Given the first several terms of a sequence in pictorial form, growing in either one or two dimensions, they can find specified terms to extend the sequence.

75th Percentile: 555

Content Area: Fractions and Number Sense

Description: Shades squares in a rectangular grid to represent a given fraction.

Shade in $\frac{3}{8}$ of the unit squares in the grid.



$$8 \overline{)24}$$

$$3 \times 3 = 9$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	89 (1.7)	▲
Hong Kong, SAR [†]	87 (1.7)	▲
Belgium (Flemish) [†]	87 (1.8)	▲
Korea, Rep. of	81 (1.4)	▲
Chinese Taipei	80 (1.9)	▲
Japan	78 (1.9)	▲
First in the World Consort., IL	71 (5.6)	▲
Canada	68 (2.6)	▲
Naperville Sch. Dist. #203, MI	67 (3.6)	▲
Michigan Invitational Group, MI	65 (5.0)	▲
Netherlands [†]	61 (4.7)	●
Fremont/Lincoln/WestSide PS, NE	59 (5.2)	●
Massachusetts	59 (3.1)	▲
Montgomery County, MD²	59 (4.7)	●
Texas	58 (4.6)	●
Academy School Dist. #20, CO	57 (4.2)	●
Indiana [†]	55 (4.9)	●
Michigan	54 (3.8)	●
Pennsylvania	53 (4.0)	●
England [†]	52 (2.9)	●
Russian Federation	52 (3.2)	●
Connecticut	52 (5.6)	●
Guilford County, NC²	51 (4.8)	●
Project SMART Consortium, OH	51 (5.6)	●
Illinois	50 (4.2)	●
Oregon	49 (3.2)	●
SW Math/Sci. Collaborative, PA	49 (3.7)	●
United States	49 (1.9)	●
Missouri	47 (4.2)	●
Idaho	46 (4.1)	●
Italy	46 (2.6)	●
North Carolina	44 (4.5)	●
Delaware Science Coalition, DE	43 (5.4)	●
South Carolina	43 (3.3)	●
Czech Republic	42 (3.2)	●
Maryland	42 (4.1)	●
Jersey City Public Schools, NJ	38 (4.1)	●
Chicago Public Schools, IL	37 (3.8)	●
Rochester City Sch. Dist., NY	32 (5.0)	▼
Miami-Dade County PS, FL	20 (3.6)	▼
International Avg. (All Countries)	49 (0.4)	
Participant average significantly higher than international average		▲
No statistically significant difference between participant average and international average		●
Participant average significantly lower than international average		▼

Significance tests adjusted for multiple comparisons

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Fractions and Number Sense

Description: Solves a multi-step word problem that involves dividing a quantity in a given ratio.

John sold 60 magazines and Mark sold 80 magazines. The magazines were all sold for the same price. The total amount of money received for the magazines was \$700. How much money did Mark receive?

Answer: \$ 400

$$\begin{array}{r}
 80 \\
 +60 \\
 \hline
 140 \text{ TOTAL}
 \end{array}$$

$$\text{MARK} = \frac{80}{140} = \frac{8}{14} = \frac{4}{7}$$

$$\begin{array}{r}
 100 \\
 7 \overline{) 700} \\
 \underline{700} \\
 0
 \end{array}$$

$$\begin{array}{r}
 100 \\
 \times 4 \\
 \hline
 \$400
 \end{array}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct
Singapore	84 (2.0) ▲
Chinese Taipei	75 (1.8) ▲
Hong Kong, SAR †	72 (2.1) ▲
Korea, Rep. of	69 (1.4) ▲
Japan	67 (2.0) ▲
Belgium (Flemish) †	60 (3.7) ▲
First in the World Consort., IL	55 (6.1) ●
Montgomery County, MD ²	54 (4.1) ●
Czech Republic	54 (3.8) ●
Netherlands †	53 (4.5) ●
Russian Federation	52 (3.1) ●
Naperville Sch. Dist. #203, IL	49 (3.9) ●
Massachusetts	46 (4.0) ●
Canada	46 (2.4) ●
Illinois	44 (2.5) ●
Oregon	43 (4.2) ●
Texas	42 (4.7) ●
South Carolina	42 (3.0) ●
SW Math/Sci. Collaborative, PA	42 (2.8) ●
Michigan	42 (3.0) ●
Jersey City Public Schools, NJ	41 (5.6) ●
Academy School Dist. #20, CO	41 (4.1) ●
United States	41 (2.0) ●
Indiana †	40 (4.4) ●
Pennsylvania	39 (3.5) ●
Guilford County, NC ²	38 (4.8) ●
Connecticut	38 (4.3) ●
North Carolina	36 (3.8) ●
Italy	36 (2.6) ●
Fremont/Lincoln/WestSide PS, NE	36 (6.5) ●
Missouri	35 (4.6) ●
Idaho	35 (3.0) ●
Delaware Science Coalition, DE	34 (5.0) ●
Project SMART Consortium, OH	34 (4.3) ●
Maryland	33 (2.4) ▼
Michigan Invitational Group, MI	32 (2.9) ▼
Chicago Public Schools, IL	32 (3.8) ▼
England †	31 (2.6) ▼
Rochester City Sch. Dist., NY	19 (3.1) ▼
Miami-Dade County PS, FL	18 (4.1) ▼
International Avg. (All Countries)	44 (0.4)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

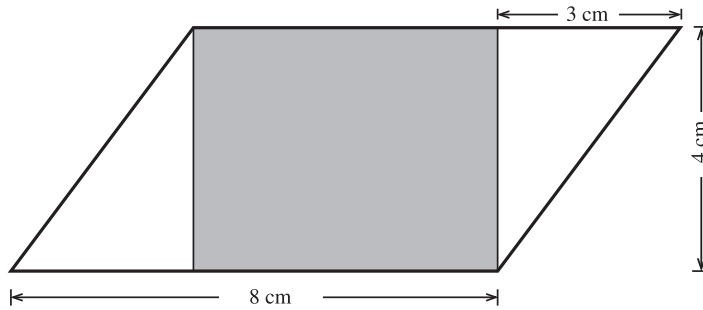
² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Measurement

Description: Finds the area of a rectangle contained in a parallelogram of given dimensions.

The figure shows a shaded rectangle inside a parallelogram.



What is the area of the shaded rectangle?

Answer: 20

$$8 - 3 = 5$$

$$\begin{array}{r} 5 \\ \times 4 \\ \hline 20 \end{array}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	83 (1.5) ▲	
Japan	80 (1.2) ▲	
Hong Kong, SAR †	78 (1.6) ▲	
Korea, Rep. of	78 (1.3) ▲	
Chinese Taipei	75 (1.4) ▲	
Naperville Sch. Dist. #203, IL	65 (2.8) ▲	
Belgium (Flemish) †	65 (2.0) ▲	
First in the World Consort., IL	62 (4.3) ▲	
Canada	58 (1.6) ▲	
Netherlands †	55 (4.7) ●	
Academy School Dist. #20, CO	49 (3.4) ●	
Russian Federation	49 (2.8) ●	
Italy	48 (2.1) ●	
England †	48 (2.3) ●	
Czech Republic	46 (2.9) ●	
Oregon	46 (4.0) ●	
Michigan Invitational Group, MI	46 (3.9) ●	
Montgomery County, MD ²	45 (3.9) ●	
Project SMART Consortium, OH	44 (4.5) ●	
Massachusetts	44 (2.8) ●	
Illinois	41 (2.9) ●	
Idaho	41 (3.8) ●	
Connecticut	40 (4.2) ●	
SW Math/Sci. Collaborative, PA	40 (3.6) ●	
Texas	40 (4.1) ●	
Michigan	39 (2.9) ●	
Fremont/Lincoln/WestSide PS, NE	38 (3.5) ●	
Indiana †	38 (3.9) ●	
Pennsylvania	34 (2.9) ▼	
Maryland	34 (2.5) ▼	
Guilford County, NC ²	34 (4.6) ●	
United States	34 (1.4) ▼	
North Carolina	33 (2.9) ▼	
South Carolina	32 (3.2) ▼	
Missouri	30 (2.5) ▼	
Delaware Science Coalition, DE	24 (3.6) ▼	
Jersey City Public Schools, NJ	22 (4.1) ▼	
Chicago Public Schools, IL	18 (4.4) ▼	
Miami-Dade County PS, FL	14 (2.4) ▼	
Rochester City Sch. Dist., NY	12 (1.9) ▼	
International Avg. (All Countries)	43 (0.3)	

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

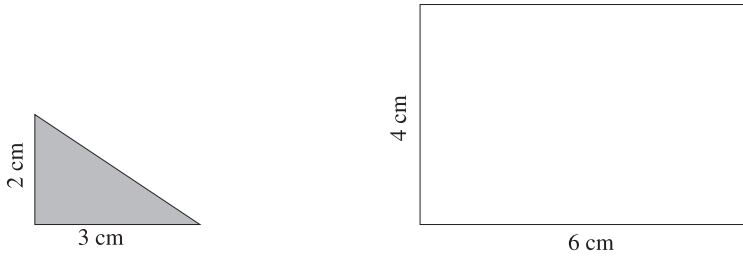
SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Geometry

Description: Determines the number of triangles of given dimensions needed to cover a given rectangle.



How many of the shaded right triangles shown above are needed to exactly cover the surface of the rectangle?

- A. Four
- B. Six
- C. Eight
- D. Ten

	Overall Percent Correct
Japan	80 (1.8) ▲
Korea, Rep. of	76 (1.7) ▲
Hong Kong, SAR †	75 (2.0) ▲
Singapore	72 (2.2) ▲
Belgium (Flemish) †	68 (2.7) ▲
Netherlands †	66 (3.8) ▲
Naperville Sch. Dist. #203, IL	62 (3.5) ▲
Chinese Taipei	60 (1.8) ▲
Michigan Invitational Group, MI	57 (6.8) ●
Montgomery County, MD ²	57 (3.9) ●
Guilford County, NC ²	56 (5.0) ●
First in the World Consort., IL	56 (5.1) ●
Czech Republic	55 (3.6) ●
South Carolina	53 (2.9) ●
Michigan	52 (3.7) ●
Oregon	50 (4.0) ●
Canada	50 (2.4) ●
Texas	50 (3.3) ●
Italy	49 (2.7) ●
England †	48 (2.6) ●
Academy School Dist. #20, CO	48 (4.3) ●
United States	47 (2.0) ●
Indiana †	47 (2.4) ●
Delaware Science Coalition, DE	46 (4.2) ●
Idaho	46 (3.7) ●
Illinois	44 (2.5) ●
Connecticut	44 (4.1) ●
SW Math/Sci. Collaborative, PA	44 (2.8) ●
Russian Federation	44 (2.8) ●
Project SMART Consortium, OH	44 (4.3) ●
Pennsylvania	42 (3.1) ●
Fremont/Lincoln/WestSide PS, NE	42 (5.5) ●
North Carolina	42 (3.2) ●
Massachusetts	41 (2.0) ●
Chicago Public Schools, IL	40 (5.4) ●
Missouri	39 (2.6) ●
Jersey City Public Schools, NJ	39 (4.4) ●
Maryland	38 (2.8) ●
Rochester City Sch. Dist., NY	31 (5.2) ●
Miami-Dade County PS, FL	30 (4.4) ▼
International Avg. (All Countries)	46 (0.4)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Algebra

Description: Solves a linear equation involving transposing.

Find the value of x if $12x - 10 = 6x + 32$

Answer: 7

$$\begin{aligned}
 12x - 6x - 10 &= 32 \\
 \Rightarrow 6x &= 42 \\
 \Rightarrow \frac{6x}{6} &= \frac{42}{6} \\
 \Rightarrow x &= 7.
 \end{aligned}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Japan	85 (1.4) ▲	
Hong Kong, SAR †	80 (1.9) ▲	
Korea, Rep. of	80 (1.5) ▲	
Russian Federation	77 (3.1) ▲	
Singapore	75 (2.8) ▲	
Chinese Taipei	73 (2.0) ▲	
Naperville Sch. Dist. #203, IL	72 (3.6) ▲	
Czech Republic	66 (2.8) ▲	
First in the World Consort., IL	61 (5.0) ▲	
Belgium (Flemish) †	58 (1.9) ▲	
Academy School Dist. #20, CO	57 (5.1) ●	
Montgomery County, MD ²	55 (4.1) ●	
Italy	46 (2.8) ●	
Indiana †	44 (5.7) ●	
Michigan	40 (3.7) ●	
Guilford County, NC ²	40 (6.1) ●	
Massachusetts	39 (3.7) ●	
South Carolina	39 (3.9) ●	
Texas	38 (5.3) ●	
SW Math/Sci. Collaborative, PA	38 (3.8) ●	
Oregon	37 (3.9) ●	
Maryland	35 (3.7) ●	
Idaho	34 (5.2) ●	
United States	34 (1.8) ▼	
Michigan Invitational Group, MI	33 (6.5) ●	
Canada	33 (3.1) ▼	
Project SMART Consortium, OH	32 (5.3) ●	
Connecticut	32 (3.7) ▼	
Illinois	32 (4.1) ▼	
Pennsylvania	31 (2.6) ▼	
North Carolina	27 (3.9) ▼	
Rochester City Sch. Dist., NY	27 (5.2) ▼	
Jersey City Public Schools, NJ	26 (5.0) ▼	
England †	26 (2.7) ▼	
Delaware Science Coalition, DE	25 (5.2) ▼	
Missouri	24 (3.1) ▼	
Fremont/Lincoln/WestSide PS, NE	22 (4.1) ▼	
Netherlands †	19 (2.9) ▼	
Miami-Dade County PS, FL	17 (4.8) ▼	
Chicago Public Schools, IL	10 (2.3) ▼	
International Avg. (All Countries)	44 (0.4)	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

- Participant average significantly higher than international average ▲
- No statistically significant difference between participant average and international average ●
- Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).


² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Achievement at the Median Benchmark

Students at the Median Benchmark demonstrated the ability to apply basic mathematical knowledge in straightforward situations (see Exhibit 2.12). For example, as shown by Example Item 10 in Exhibit 2.13, students showed that they understand rounding and can use it to estimate the results of computations. Given the number of rows of cars in a parking lot and the number of cars in each row, students chose the number sentence that would give the best estimate of the total number of cars. While students at the Lower Quarter Benchmark rounded to the nearest hundred, students at the Median Benchmark successfully rounded numbers to get the best estimate for a product. Moreover, middle-performing students demonstrated greater competence with word problems than did those at the Lower Quarter Benchmark. The Benchmarking participants performed particularly well on this test question involving rounding. The international average percent correct for this item was 65 percent, and all except five Benchmarking entities performed significantly above the international average. Among the high-achieving countries, Singapore outperformed other countries with 94 percent correct, followed by 85 percent in Hong Kong. More than 85 percent of students answered correctly in Naperville, the First in the World Consortium, Guilford County, the Academy School District, the Southwest Pennsylvania Math and Science Collaborative, Indiana, North Carolina, and Connecticut.

In geometry, students at the Median Benchmark were able to locate a point on a grid with five-unit divisions that lies between the grid lines (see Example Item 11 in Exhibit 2.14). Fifty-eight percent of students on average internationally correctly chose Point S as the point on the grid that could have the coordinates (7,16). In Japan, Korea, Chinese Taipei, Hong Kong, and Singapore, 80 percent or more of the students answered correctly, as did students in Naperville and First in the World. Generally, the Benchmarking participants performed relatively well on this question, with 13 of them performing significantly above the international average. As might be anticipated, students answering incorrectly most commonly chose Point Q (16,7).

Example Item 12 shown in Exhibit 2.15 illustrates students' emerging familiarity with algebraic representation. Internationally on average, nearly two-thirds of students correctly identified the linear equation corresponding to a given verbal statement involving a variable. In Hong Kong, Singapore, Japan, and Korea, 85 percent or more of the students answered correctly, and eighth graders in several Benchmarking



districts and consortia performed similarly. Naperville (94 percent) topped the chart on this item, and 85 percent or more of the students in the First in the World Consortium, Montgomery County, and the Academy School District answered correctly.

• Median Benchmark**Summary**

Students can apply basic mathematical knowledge in straightforward situations. They can add or subtract to solve one-step word problems involving whole numbers and decimals; identify representations of common fractions and relative sizes of fractions; solve for missing terms in proportions; recognize basic notions of percents and probability; use basic properties of geometric figures; read and interpret graphs, tables, and scales; and understand simple algebraic relationships.

Students can apply basic mathematical knowledge in straightforward situations. They are able to use addition and subtraction to solve one-step word problems involving whole numbers and decimals. They can round whole numbers to the nearest hundred and identify the number sentence that gives the best estimate for the product of two numbers after rounding. Students can arrange four given digits in descending and ascending order to form the largest and smallest possible numbers, and find the difference between those two numbers. Students can approximate the quantity remaining after an amount is reduced by a given percent.

Students demonstrate an understanding of place value in decimal numbers. They can estimate the location of a point representing a decimal number in tenths on a number line marked in whole numbers and identify an unlabeled midway point on a number line marked in tenths. They can set up and solve one-step problems involving addition and subtraction of numbers having up to three decimal places, including situations where the numbers have a different number of decimal places. Given an object of one length, to one decimal place, they can estimate the length of another object.

Students can select the smallest fraction from a list of fractions and can recognize models representing fractions as shaded regions. They can find the missing term in a proportion in word problems and number sentences. Students can solve a simple word problem involving the likelihood of a successful outcome.

Students are able to select the appropriate metric unit to measure the mass of an object. They recognize the inverse relationship between the length of a unit and the number of units required to cover a distance.

Students can locate and interpret data presented in bar graphs, pictographs, pie graphs, and line graphs. Given a table of values for two variables, they can select the graph that represents the given data.

Students can solve problems involving the properties of congruent figures and can select a pair of similar triangles from a set of triangles. They can visualize a rotation of a three-dimensional figure made of cubes. They can locate points in the first quadrant of the Cartesian plane.

Students can select an expression to represent a situation involving multiplication, and identify a linear equation corresponding to a verbal statement. They can find a missing value in a table of values relating x and y values. Using the properties of a balance, they can reason to find an unknown weight. Given diagrams representing the first few terms of a sequence, growing in one dimension, and a partially completed table, they can find the next two terms.

50th Percentile: 479

Content Area: Fractions and Number Sense

Description: In a word problem, uses rounding to identify the number sentence that gives the best estimate for the product.

There are 68 rows of cars in a parking lot. Each row has 92 cars. Which of these would give the closest estimate of the total number of cars in the parking lot?

- A. $60 \times 90 = 5400$
- B. $60 \times 100 = 6000$
- C. $70 \times 90 = 6300$**
- D. $70 \times 100 = 7000$

	Overall Percent Correct	
Naperville Sch. Dist. #203, IL	95 (2.1)	▲
Singapore	94 (1.0)	▲
First in the World Consort., IL	93 (3.2)	▲
Guilford County, NC ²	87 (3.4)	▲
Academy School Dist. #20, CO	87 (3.0)	▲
SW Math/Sci. Collaborative, PA	87 (3.1)	▲
Indiana [†]	86 (2.6)	▲
North Carolina	86 (1.9)	▲
Connecticut	86 (3.6)	▲
Michigan Invitational Group, MI	85 (3.8)	▲
Illinois	85 (2.2)	▲
Hong Kong, SAR [†]	85 (1.7)	▲
Montgomery County, MD ²	85 (3.2)	▲
Michigan	85 (2.6)	▲
Chicago Public Schools, IL	84 (2.1)	▲
Oregon	84 (2.1)	▲
Belgium (Flemish) [†]	83 (3.0)	▲
Japan	82 (1.4)	▲
Korea, Rep. of	82 (1.2)	▲
Chinese Taipei	81 (1.5)	▲
South Carolina	81 (2.9)	▲
Texas	81 (3.5)	▲
Netherlands [†]	81 (3.1)	▲
Idaho	81 (3.6)	▲
Pennsylvania	80 (3.9)	▲
Project SMART Consortium, OH	80 (4.7)	▲
United States	79 (1.8)	▲
Canada	78 (2.1)	▲
Czech Republic	78 (2.3)	▲
Massachusetts	76 (2.8)	▲
Missouri	75 (2.6)	▲
Fremont/Lincoln/WestSide PS, NE	75 (4.0)	●
Delaware Science Coalition, DE	74 (3.2)	●
England [†]	74 (2.8)	▲
Maryland	74 (1.9)	▲
Jersey City Public Schools, NJ	71 (3.2)	●
Rochester City Sch. Dist., NY	67 (3.8)	●
Russian Federation	65 (2.7)	●
Miami-Dade County PS, FL	60 (3.5)	●
Italy	52 (2.5)	▼
International Avg. (All Countries)	65 (0.4)	

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

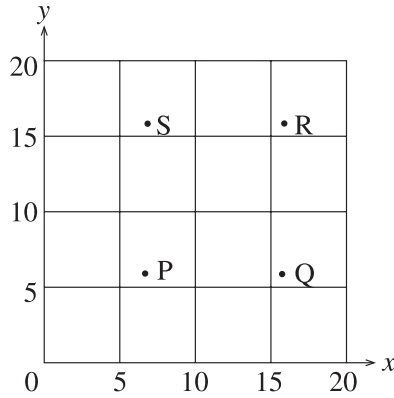
* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Geometry

Description: Locates the point on a grid with 5-unit divisions when the point lies between the grid lines.

Which point on the graph could have coordinates (7,16)?



- A. Point P
- B. Point Q
- C. Point R
- D. Point S**

	Overall Percent Correct
Naperville Sch. Dist. #203, IL	88 (2.9) ▲
Japan	84 (1.7) ▲
Korea, Rep. of	84 (1.4) ▲
Chinese Taipei	83 (1.5) ▲
First in the World Consort., IL	82 (3.2) ▲
Hong Kong, SAR [†]	81 (1.7) ▲
Singapore	80 (2.3) ▲
Netherlands [†]	78 (2.5) ▲
North Carolina	78 (3.2) ▲
Jersey City Public Schools, NJ	76 (4.4) ▲
Guilford County, NC²	75 (4.2) ▲
England [†]	75 (3.2) ▲
SW Math/Sci. Collaborative, PA	74 (3.3) ▲
Texas	74 (3.4) ▲
South Carolina	73 (3.5) ▲
Academy School Dist. #20, CO	73 (3.3) ▲
Montgomery County, MD²	73 (3.0) ▲
Michigan	72 (2.9) ▲
Pennsylvania	71 (2.0) ▲
Russian Federation	71 (2.2) ▲
Belgium (Flemish) [†]	71 (2.5) ▲
Oregon	70 (5.3) ●
Michigan Invitational Group, MI	69 (3.8) ●
Illinois	69 (3.3) ▲
Project SMART Consortium, OH	68 (4.8) ●
Canada	67 (2.6) ▲
Indiana[†]	67 (3.2) ●
United States	67 (1.6) ▲
Maryland	67 (3.7) ●
Massachusetts	64 (3.0) ●
Italy	62 (2.2) ●
Connecticut	61 (4.5) ●
Delaware Science Coalition, DE	60 (4.6) ●
Missouri	60 (3.0) ●
Czech Republic	58 (3.2) ●
Chicago Public Schools, IL	57 (5.3) ●
Miami-Dade County PS, FL	56 (4.2) ●
Idaho	56 (5.8) ●
Rochester City Sch. Dist., NY	55 (4.7) ●
Fremont/Lincoln/WestSide PS, NE	54 (6.1) ●
International Avg. (All Countries)	58 (0.4)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.



Content Area: Algebra

Description: Identifies the linear equation corresponding to a given verbal statement involving a variable.

n is a number. When n is multiplied by 7, and 6 is then added, the result is 41. Which of these equations represents this relation?

- A. $7n + 6 = 41$
- B. $7n - 6 = 41$
- C. $7n \times 6 = 41$
- D. $7(n + 6) = 41$

	Overall Percent Correct
Naperville Sch. Dist. #203, IL	94 (1.4) ▲
Hong Kong, SAR [†]	93 (0.9) ▲
First in the World Consort., IL	90 (1.4) ▲
Singapore	89 (1.7) ▲
Montgomery County, MD ²	87 (1.4) ▲
Japan	86 (0.8) ▲
Academy School Dist. #20, CO	85 (1.6) ▲
Korea, Rep. of	85 (0.7) ▲
Chinese Taipei	84 (1.1) ▲
Michigan	82 (1.6) ▲
Canada	82 (1.0) ▲
Russian Federation	82 (1.6) ▲
Project SMART Consortium, OH	82 (2.1) ▲
Pennsylvania	81 (1.8) ▲
Belgium (Flemish) [†]	81 (1.2) ▲
Massachusetts	80 (2.1) ▲
Netherlands [†]	80 (2.5) ▲
Connecticut	80 (2.7) ▲
Indiana [†]	80 (2.3) ▲
Michigan Invitational Group, MI	80 (2.3) ▲
SW Math/Sci. Collaborative, PA	80 (2.5) ▲
Illinois	80 (2.3) ▲
Guilford County, NC ²	79 (2.3) ▲
Texas	78 (3.6) ▲
United States	77 (1.3) ▲
Oregon	77 (2.0) ▲
Idaho	77 (2.7) ▲
South Carolina	77 (2.3) ▲
North Carolina	75 (2.4) ▲
Missouri	73 (1.7) ▲
Maryland	72 (2.6) ●
Fremont/Lincoln/WestSide PS, NE	72 (4.1) ●
Czech Republic	72 (1.7) ▲
Delaware Science Coalition, DE	71 (3.3) ●
Chicago Public Schools, IL	71 (2.3) ●
Jersey City Public Schools, NJ	69 (2.6) ●
Rochester City Sch. Dist., NY	68 (3.4) ●
Miami-Dade County PS, FL	64 (2.3) ●
England [†]	62 (2.1) ●
Italy	58 (1.6) ▼
International Avg. (All Countries)	65 (0.3)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).


² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Achievement at the Lower Quarter Benchmark

As shown in Exhibit 2.16, the few items anchoring at the Lower Quarter Benchmark provided evidence that students performing at this level can add, subtract, and round with whole numbers. For example, students answering Example Item 13 correctly rounded 691 and 208 to estimate their sum as close to the sum of 700 and 200 (see Exhibit 2.17). The international average was 80 percent correct, and 27 countries had three-quarters or more of their students choosing the correct answer. In four countries – Singapore, Belgium (Flemish), Japan, and the Netherlands – 95 percent or more of the students gave the correct response. That level of performance was attained by students in twelve Benchmarking entities: Naperville, Indiana, the Michigan Invitational Group, the Southwest Pennsylvania Math and Science Collaborative, Montgomery County, the Project SMART Consortium, Connecticut, Pennsylvania, Illinois, Missouri, Texas, and the First in the World Consortium. Again, the Benchmarking participants did comparatively well on this rounding item. In all, students in every Benchmarking entity except the Miami-Dade County Public Schools achieved significantly above the international average.

As illustrated by Example Item 14 in Exhibit 2.18, students at the Lower Quarter Benchmark generally could subtract one three-decimal-place number from another with multiple regrouping. Internationally on average, 77 percent of the eighth-grade students selected the correct response to this item. Students in Texas (89 percent) performed significantly above the international average and similarly to students in Singapore, Korea, and the Russian Federation (88 to 90 percent). All of the other Benchmarking participants performed near the international average except the Michigan Invitational Group (60 percent), whose students performed below it.

Students at this level could subtract one four-digit integer from another involving multiple regrouping with zeroes (see Example Item 15 in Exhibit 2.19). On this subtraction item also, students in Texas (90 percent) performed similarly to those in Singapore, Chinese Taipei, and Hong Kong (90 to 92 percent). Students in the Naperville School District (88 percent), the Academy School District (84 percent), and Massachusetts (82 percent) also performed significantly above the international average of 74 percent.



In addition, Example Item 16 in Exhibit 2.20 shows that students at this level could read a thermometer and locate the correct reading in a table. Internationally on average, 79 percent of students answered the item correctly. Students in the Benchmarking entities performed comparatively well on this question. Sixteen of the Benchmarking participants performed significantly above the international average and none below it. Essentially all of the students in Naperville (99 percent) responded correctly, and 90 percent or more did so in First in the World, the Academy School District, Illinois, Project SMART, Indiana, the Southwest Pennsylvania Math and Science Collaborative, and Massachusetts.

• Lower Quarter Benchmark

Summary

Students can do basic computations with whole numbers.

The few items at this level provide some evidence that students can add, subtract, and round with whole numbers. When there are the same number of decimal places, they can subtract with multiple regrouping. Students can round whole numbers to the nearest hundred. They can read a thermometer and locate the reading in a table. Students recognize some basic notation.

25th Percentile: 396

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Content Area: Fractions and Number Sense

Description: Rounds to estimate the sum of two three-digit numbers.

The sum $691 + 208$ is closest to the sum

- A. $600 + 200$
- B. $700 + 200$**
- C. $700 + 300$
- D. $900 + 200$

	Overall Percent Correct	
Naperville Sch. Dist. #203, IL	99 (0.5)	▲
Indiana [†]	97 (0.7)	▲
Singapore	97 (0.5)	▲
Michigan Invitational Group, MI	97 (1.0)	▲
Belgium (Flemish) [†]	96 (0.7)	▲
SW Math/Sci. Collaborative, PA	96 (1.1)	▲
Montgomery County, MD ²	95 (0.9)	▲
Project SMART Consortium, OH	95 (1.0)	▲
Japan	95 (0.5)	▲
Connecticut	95 (1.1)	▲
<i>Pennsylvania</i>	95 (1.1)	▲
Illinois	95 (0.9)	▲
Missouri	95 (0.8)	▲
Netherlands [†]	95 (0.8)	▲
Texas	95 (0.8)	▲
First in the World Consort., IL	95 (1.6)	▲
Academy School Dist. #20, CO	94 (1.2)	▲
Michigan	94 (1.0)	▲
North Carolina	94 (1.2)	▲
Oregon	94 (1.2)	▲
Idaho	93 (1.2)	▲
Massachusetts	93 (0.8)	▲
Hong Kong, SAR [†]	93 (0.7)	▲
Jersey City Public Schools, NJ	93 (1.5)	▲
Canada	93 (0.7)	▲
United States	93 (0.7)	▲
South Carolina	93 (1.1)	▲
Maryland	93 (1.0)	▲
Chicago Public Schools, IL	93 (1.6)	▲
Korea, Rep. of	93 (0.6)	▲
England [†]	92 (1.0)	▲
Guilford County, NC ²	92 (1.6)	▲
Delaware Science Coalition, DE	92 (1.4)	▲
Czech Republic	91 (1.0)	▲
Chinese Taipei	89 (0.7)	▲
Fremont/Lincoln/WestSide PS, NE	89 (1.5)	▲
Rochester City Sch. Dist., NY	88 (2.1)	▲
Russian Federation	83 (1.9)	●
Miami-Dade County PS, FL	83 (3.2)	●
Italy	77 (1.9)	●
International Avg. (All Countries)	80 (0.2)	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

* The item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
[†] Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Fractions and Number Sense

Description: Subtracts a three-decimal-place number from another with multiple regrouping.

Subtract: $4.722 - 1.935 =$

- A.** 2.787
- B. 2.797
- C. 2.887
- D. 2.897

	Overall Percent Correct
Singapore	90 (1.4) ▲
<i>Texas</i>	89 (2.1) ▲
Korea, Rep. of	88 (1.2) ▲
Russian Federation	88 (1.9) ▲
Japan	86 (1.3) ▲
Czech Republic	85 (2.8) ●
Chinese Taipei	84 (1.5) ▲
Naperville Sch. Dist. #203, IL	84 (2.9) ●
Chicago Public Schools, IL	83 (2.8) ●
Hong Kong, SAR †	83 (1.8) ▲
Indiana †	82 (2.7) ●
Montgomery County, MD †	82 (3.4) ●
South Carolina	81 (2.6) ●
Academy School Dist. #20, CO	81 (3.3) ●
Canada	80 (1.8) ●
Illinois	78 (2.2) ●
Guilford County, NC †	78 (4.0) ●
Pennsylvania	78 (2.8) ●
Project SMART Consortium, OH	78 (3.3) ●
Rochester City Sch. Dist., NY	77 (3.9) ●
Massachusetts	77 (2.6) ●
Maryland	77 (2.2) ●
United States	77 (1.7) ●
Italy	77 (2.3) ●
Connecticut	77 (4.0) ●
SW Math/Sci. Collaborative, PA	76 (3.4) ●
Jersey City Public Schools, NJ	76 (5.2) ●
North Carolina	76 (2.6) ●
Idaho	75 (3.9) ●
Michigan	74 (3.1) ●
Oregon	73 (3.6) ●
Belgium (Flemish) †	73 (2.0) ●
First in the World Consort., IL	73 (3.5) ●
Miami-Dade County PS, FL	71 (4.0) ●
Netherlands †	69 (4.3) ●
Missouri	68 (4.2) ●
Delaware Science Coalition, DE	68 (3.5) ●
Fremont/Lincoln/WestSide PS, NE	61 (5.6) ●
Michigan Invitational Group, MI	60 (4.4) ▼
England †	59 (2.7) ▼
International Avg. (All Countries)	77 (0.4)

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

* The item was answered correctly by a majority of students reaching this benchmark.

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Fractions and Number Sense

Description: Subtracts a four-digit number from another involving zeroes.

Subtract:

$$\begin{array}{r} 7003 \\ - 4078 \\ \hline \end{array}$$

- A. 2035
- B. 2925**
- C. 3005
- D. 3925

	Overall Percent Correct	
Singapore	92 (1.3)	▲
Chinese Taipei	90 (1.2)	▲
Texas	90 (1.9)	▲
Hong Kong, SAR †	90 (1.3)	▲
Korea, Rep. of	88 (1.2)	▲
Naperville Sch. Dist. #203, IL	88 (2.7)	▲
Japan	86 (1.4)	▲
Belgium (Flemish) †	85 (2.1)	▲
Academy School Dist. #20, CO	84 (2.8)	▲
Indiana †	84 (3.3)	●
Canada	83 (1.4)	▲
Massachusetts	82 (2.3)	▲
Montgomery County, MD ²	82 (4.3)	●
Illinois	82 (2.4)	●
Czech Republic	82 (2.4)	●
Jersey City Public Schools, NJ	81 (2.8)	●
Idaho	81 (2.8)	●
United States	81 (1.6)	▲
Oregon	80 (2.1)	●
Guilford County, NC ²	80 (4.5)	●
Chicago Public Schools, IL	80 (4.9)	●
Russian Federation	79 (2.2)	●
Netherlands †	79 (3.4)	●
SW Math/Sci. Collaborative, PA	79 (2.9)	●
Michigan Invitational Group, MI	78 (4.8)	●
Missouri	77 (3.3)	●
Pennsylvania	77 (2.4)	●
Connecticut	77 (3.8)	●
South Carolina	77 (2.6)	●
Project SMART Consortium, OH	76 (4.0)	●
Rochester City Sch. Dist., NY	76 (4.5)	●
Maryland	76 (2.2)	●
North Carolina	75 (2.7)	●
Delaware Science Coalition, DE	75 (3.2)	●
First in the World Consort., IL	74 (4.0)	●
Michigan	73 (3.2)	●
Miami-Dade County PS, FL	72 (2.8)	●
Fremont/Lincoln/WestSide PS, NE	68 (6.4)	●
Italy	67 (2.7)	●
England †	51 (3.1)	▼
International Avg. (All Countries)	74 (0.4)	

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Participant average significantly higher than international average ▲

No statistically significant difference between participant average and international average ●

Participant average significantly lower than international average ▼

Significance tests adjusted for multiple comparisons

* This item was answered correctly by a majority of students reaching this benchmark.
States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

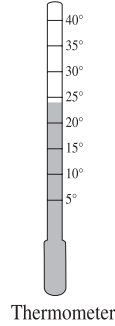
² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Content Area: Data Representation, Analysis and Probability

Description: Reads a thermometer and locates the reading in a table.

This table shows temperatures at various times on four days.

TEMPERATURE					
	6 a.m.	9 a.m.	Noon	3 p.m.	6 p.m.
Monday	15°	17°	24°	21°	16°
Tuesday	20°	16°	15°	10°	9°
Wednesday	8°	14°	16°	19°	15°
Thursday	8°	11°	19°	26°	20°



On which day and at what time was the temperature shown in the table the same as that shown on the thermometer.

- A. Monday, Noon
- B. Tuesday, 6 a.m.
- C. Wednesday, 3 p.m.
- D. Thursday, 3 p.m.

	Overall Percent Correct
Naperville Sch. Dist. #203, IL	99 (1.0) ▲
Japan	96 (0.8) ▲
Singapore	95 (0.9) ▲
Belgium (Flemish) †	95 (1.5) ▲
First in the World Consort., IL	95 (2.7) ▲
Academy School Dist. #20, CO	92 (2.1) ▲
Korea, Rep. of	92 (0.9) ▲
England †	92 (2.2) ▲
Chinese Taipei	91 (1.2) ▲
Czech Republic	91 (1.9) ▲
Illinois	91 (1.8) ▲
Project SMART Consortium, OH	91 (3.7) ●
Indiana †	91 (1.9) ▲
SW Math/Sci. Collaborative, PA	91 (1.8) ▲
Hong Kong, SAR †	90 (1.5) ▲
Netherlands †	90 (2.6) ▲
Massachusetts	90 (2.0) ▲
Canada	89 (2.6) ▲
United States	89 (1.2) ▲
Fremont/Lincoln/WestSide PS, NE	89 (2.2) ▲
Montgomery County, MD ²	89 (3.2) ●
North Carolina	89 (2.2) ▲
Idaho	89 (2.6) ▲
Oregon	88 (1.9) ▲
Michigan Invitational Group, MI	88 (3.3) ●
Texas	88 (2.3) ▲
Guilford County, NC ²	88 (4.1) ●
Michigan	88 (2.7) ▲
Pennsylvania	87 (3.6) ●
Connecticut	87 (3.6) ●
Missouri	87 (1.9) ▲
Maryland	87 (1.8) ▲
Delaware Science Coalition, DE	87 (3.2) ●
South Carolina	87 (2.1) ▲
Chicago Public Schools, IL	86 (3.5) ●
Russian Federation	85 (2.6) ●
Italy	81 (2.0) ●
Jersey City Public Schools, NJ	81 (2.1) ●
Miami-Dade County PS, FL	76 (5.2) ●
Rochester City Sch. Dist., NY	73 (4.7) ●
International Avg. (All Countries)	79 (0.3)
Participant average significantly higher than international average	▲
No statistically significant difference between participant average and international average	●
Participant average significantly lower than international average	▼
Significance tests adjusted for multiple comparisons	

* This item was answered correctly by a majority of students reaching this benchmark.
 States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).
 † Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

² National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).
 () Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

What Issues Emerge from the Benchmark Descriptions?

The benchmark descriptions and example items strongly suggest a gradation in achievement, from the top-performing students' ability to generalize and solve non-routine or contextualized problems to the lower-performing students being able primarily to use routine, mainly numeric procedures. The fact that even at the Median Benchmark students demonstrate only limited achievement in problem solving beyond straightforward one-step problems may suggest a need to reconsider the role, or priority, of problem solving in mathematics curricula.

The choices teachers make determine, to a large extent, what students learn. According to the NCTM's "The Teaching Principle," in effective teaching worthwhile mathematical problems are used to introduce important ideas and engage students' thinking. The TIMSS 1999 Benchmarking results show that higher achievement is related to the emphasis that teachers place on reasoning and problem-solving activities (see Chapter 6, Exhibit 6.11). This finding is consistent with the video study component of TIMSS conducted in 1995. Analyses of videotapes of mathematics classes revealed that in the typical mathematics lesson in Japan students worked on developing solution procedures to report to the class that were often expected to be original constructions. In contrast, in the typical U.S. lesson students essentially practiced procedures that had been demonstrated by the teacher.

In looking across the item-level results, it is also important to note the variation in performance across the topics covered. On the 16 items presented in this chapter, there was a substantial range in performance for many Benchmarking participants. For example, students in the Benchmarking entities performed relatively well on the items requiring rounding (Exhibits 2.13 and 2.17), and students in Texas did very well on the subtraction questions (Exhibits 2.18 and 2.19). Conversely, students in the Benchmarking entities had particular difficulty with measurement items containing figures (Exhibits 2.2 and 2.9). In some cases, differences of this sort will result from intended differences in emphasis in state or district curricula. It is likely, however, that variation in results may be unintended, and the findings will provide important information about strengths and weaknesses in intended or implemented curricula. For example, Maryland, the Michigan Invitational Group, Chicago, Rochester, and Miami-Dade may not have anticipated performing below the international average on a relatively straightforward word problem involving proportional reasoning (Exhibit 2.8). At the very least, an in-depth examination of the TIMSS 1999 results may reveal aspects of curricula that merit further investigation.